

A new proposal regarding the heat generated by gravity in locally accelerating frames

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For Rindler observers accelerating close to the horizon in local patches around a spacetime point, the matter-energy passing through the horizon increases the entropy and heat energy. Jacobson has showed that the Einstein equation can be derived from the consideration of this thermodynamic process. This, however, works only if the acceleration a is much larger than the scale set by the curvature of the spacetime. It is explored here whether an extension is possible to the case with no lower bound on a . We show that this is possible if one assumes that in a locally accelerating frame, the matter-energy passing through null hypersurfaces could result in an increase in the heat energy and the entropy. Such a generalisation extends the thermodynamic derivation of gravity to include any non-freely falling observer. A new method of determining the temperature detected by such locally accelerating observers is also presented. By considering only the quantisation of sufficiently localised wave modes of a field, it is shown that the observer finds himself in a thermal environment.

I. INTRODUCTION

There are strong reasons to believe that there is a connection between Gravity and Thermodynamics, even though, so far we do not know what the microscopic degrees of freedom are in this case. Historically, the laws of Black hole mechanics [1], which are of a form strikingly similar to that of the laws of thermodynamics, came first. Motivated by this and other considerations, Bekenstein proposed that the Black holes have a temperature and possess an entropy proportional to the surface area of the Black hole event horizon[2]. The similarity became much more physically compelling, once Hawking radiation was discovered from semiclassical considerations[3]. Around the same time, it was discovered that even in the Minkowski spacetime, an accelerating observer in a Rindler frame would encounter thermal radiation whereas an inertial observer sees only vacuum [4].

The black hole spacetime and the Rindler spacetime have one thing in common, causal horizons exist for both of them. The vacuum fluctuations in the frame of the inertial observer are seen as

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thermal fluctuations by the Rindler observer. This correspondence holds for the flat spacetime. However, it is possible to consider accelerating observers at any spacetime point and construct a locally Rindler patch around it. Such observers are expected to find themselves in a thermal environment. The temperature detected by them is proportional to the acceleration as in Rindler spacetime. One notices here an interesting similarity between observers who remain outside black holes and observers who are locally accelerating. For black holes, however, the analogy of the laws of black hole mechanics with thermodynamics was noticed before the discovery of the thermal nature of the radiation from the black hole. But it is also possible to proceed in the reverse direction and explore whether the thermal nature of the radiation encountered locally by the accelerated observer in a spacetime is accompanied by some interesting consequences of thermodynamics. A pioneering step in this direction was taken by Jacobson[5]. For this, he had to use the thermodynamic properties of a Rindler-like horizon in a local construction. So he considered an observer accelerating at a constant rate in an arbitrary spacetime in such a way that one can construct a locally Rindler patch containing the observer, with the Rindler horizon forming a part of its boundary. The amount of matter-energy that passed through this horizon was taken to be the increase in the heat energy, dQ , while the increase in the horizon area was assumed to be proportional the increase in the entropy ds of the system. Assuming this, he was able to show that the relation, $dQ = Tds$, then gives back the Einstein equation. Based on this result, it was suggested by him that gravity itself could be viewed as an emergent phenomenon [Also see discussions in [6]]. This derivation has recently been extended to include some other theories of gravity as well [7], [8], [9], [10], [11], [12].

It is clear that such a derivation of the Einstein equation has two inherent limitations. The first one is that for the inertial-Rindler observer correspondence between vacuum-thermal fluctuations to work, the spacetime has to be flat and this in general is only possible to achieve locally. That, however, does not pose any real problem. In fact, it could be thought of as an attractive feature of this derivation, where the Einstein equation emerges as a consequence of the thermodynamics at a local level.

The second limitation is essentially a restriction in the class of locally accelerating observers, for whom this derivation of the Einstein equation from thermodynamics works. It arises mainly from the consideration of what the accelerating observer is going to see and interpret as an increase in heat energy. The idea here is that the information about the degrees of freedom of a macroscopic body is lost to the observer as it reaches the horizon. That energy then appears as heat energy to the observer, who remains outside the horizon. This intuition receives strong support from other considerations like the first law of horizon thermodynamics [13] and the results obtained within

the membrane paradigm for black holes [14]. This implies that to make use of the thermodynamic correspondence, the local Rindler patch has to be constructed in such a way that the Rindler horizon forms a part of the boundary of that patch. In addition to this, there is a further constraint, which is more technical in nature. For Jacobson's derivation to work, one has to consider matter-energy passing through close to the bifurcation surface of the Rindler horizon of the local Rindler patch. The bifurcation surface of a Rindler wedge is at a distance of $\frac{1}{a}$ from the accelerating observer. Now, from the consideration of Riemann normal C.S., the size L of the local patch would be such that $L \ll \frac{1}{\sqrt{Riemann}}$. The distance of the Rindler observer from the bifurcation point should not exceed that length scale. It follows then that the acceleration should be such that, $\frac{1}{a} \ll \frac{1}{\sqrt{Riemann}}$. This condition restricts the class of locally accelerating observers, for whom the Einstein equation is a consequence of thermodynamics. Thus among all the observers, who are locally accelerating, the above derivation of Einstein equation from thermodynamics would only hold for the subclass of observers, who accelerate at a rate much higher than the scale set by the curvature of the spacetime. It is possible to take a limit where the acceleration goes to infinity and so does the boost energy, which passes through the horizon, making the analysis strictly local. In fact, this was the way, that this analysis was carried out originally by Jacobson [5]. From the thermodynamic point of view, this means that the temperature is going to infinity and so does the amount of heat generated as seen by the observer. There ratio remains finite however and gives the change in the entropy of the system. It is important to note here that one can take this limit sensibly because the amount of entropy increase is independent of the scale of the acceleration. This is a feature that is expected to persist in our generalisation of Jacobson's derivation.

The accelerating observer sees thermal radiation at a temperature proportional to a in a Rindler patch bounded on one side by a Rindler horizon. It is less clear however, whether an observer accelerating locally, for whom the Rindler horizon does not exist in a local patch, would still find herself in a thermal environment. Here we demonstrate by a semiclassical calculation that such an observer would detect a temperature proportional to the acceleration. We focus on the Fourier modes of a massless scalar field, which are localised within this patch and build up Fock states using only those modes. Then it can be shown that provided the 'inertial observer' sees a vacuum state, the accelerating observer finds the environment to be thermal with a temperature proportional to the acceleration.

Normally, if a system has a temperature, then one can reasonably expect that a thermodynamic description exists for the macroscopic processes concerning the system. This was found to be true for black holes. Jacobson's derivation of Einstein equation from thermodynamics is also a

confirmation of that expectation. From this point of view, the restriction of this derivation to a subclass of locally accelerating observers appears unsatisfactory. The locally accelerating observers who have been excluded, also detect a temperature. So a thermodynamic description should exist for all the observers, who are accelerating locally.

One can also note the work done by Verlinde[15] in this context, who claimed that gravity can be thought of as an entropic force. In particular, assuming that the relevant degrees of freedom for gravity resides on two dimensional spacelike surfaces, through which the matter passes and Equipartition theorem holds for such degrees of freedom, he was able to derive the Einstein equation, or more precisely, the time-time component of it [16]. Our approach here is different from the entropic gravity perspective and hence we would not refer to it any further. However, it is important to note that his work suggested that a thermodynamic interpretation may also be possible for observers not belonging to the class, for which Jacobson's method works.

The possibility of extending the thermodynamic derivation of the Einstein equation to all observers who are accelerating locally would be explored here. For this, one needs a prescription to determine both the increase in entropy and heat energy. To be able to do this, a geometric structure has to be identified, to which, one could assign thermodynamic properties. In the case considered by Jacobson, the Rindler horizon served this purpose. However, locally one can always find a class of accelerating observers, who see a given null hypersurface as the Rindler horizon. Moreover, as Padmanabhan and Paranjape has showed [17], one can derive the dynamical equations of gravity by assigning an entropy functional to such a null hypersurface. In fact, using the Einstein equation, it is possible to write down the dynamics of such null hypersurfaces in the form of Navier-Stokes equation [18],[19],[20],[21]. All this suggests that such null hypersurfaces are the natural choice for the generalisation sought here. Accordingly, we choose the null hypersurfaces within the local patch to determine the increase in entropy and heat. The change in entropy is assumed to be proportional to the change in the area of the cross section of the null boundary of the local patch. This may seem somewhat similar in spirit to the estimation of the degrees of freedom in a causal diamond according to the Holographic principle[22].

How to determine the increase in the amount of heat energy is less apparent, however. A proposal is made here, which leads to a prescription, that could be used to compute dQ . The main idea here is that a fraction of the matter-energy passing through a null hypersurface gets converted into heat in a locally accelerating frame. When the null surface is not a horizon, the fraction of matter-energy that gets converted would be typically very small. Two criteria have been used to arrive at such a prescription to determine dQ .

- 1) When evaluated using this prescription, $dQ = Tds$ should give the Einstein equation.
- 2) When used to compute dQ for the matter-energy passing through the horizon for a locally Rindler-like observer, the prescription should be able to produce the standard result.

The main point in our paper then is the proposal that just like matter-energy passing through the horizon becomes heat energy and increases the entropy, in an accelerating frame, matter-energy passing through the null hypersurfaces within a local patch, could also result in the increase of heat and entropy by a tiny amount.

The paper is organised as follows. In the next section, the Einstein equation is derived from thermodynamic considerations. In the third section, we show that how a person at rest in a locally accelerating frame sees a thermal environment. In the final section, the implications of this result are discussed.

II. DERIVATION OF THE EINSTEIN EQUATION FROM EQUILIBRIUM THERMODYNAMICS

We now demonstrate how the Einstein equation can be derived from thermodynamic considerations. For this, we have to choose a local neighbourhood around some spacetime point P in an arbitrary spacetime. After defining the local patch properly and then introducing an inertial and an accelerating coordinate system covering it, we can proceed with the thermodynamic analysis.

A. Construction of a local patch and a causal diamond around a spacetime point

We can now consider a causal diamond \mathcal{D} (see FIG. 1) around the point P , sufficiently small, so that one can neglect the effects of the curvature of the spacetime within it. One can construct a C.S. with a Minkowski metric in this patch. Anybody at rest or in uniform motion in this C.S. can be thought of as an Inertial Observer. Now we consider an observer moving with a constant acceleration a in this local patch. One can also construct a C.S. within this causal diamond, in which the accelerating observer is at rest. We now choose the boundaries of \mathcal{D} to be the null boundaries of the local patch, where the observer resides. This can be done in the following way. First, let us consider the local observer to be such that he or she could typically measure lengths or time differences much smaller than the size of the local patch. Suppose that the size of \mathcal{D} is of order L . Then the length scale associated with the local observers would be much smaller than L . This can be achieved by rescaling the coordinates by a factor Λ for the local observers such that

$\Lambda \gg 1$. This means that the size of \mathcal{D} is now of the order of ΛL in the rescaled units of the local observers. If one takes Λ to be sufficiently large, ΛL would also be a very large quantity. In that case, one may think of the size of the local patch to be as large as possible or practically infinity as far as the local observers are concerned. One can take only such null hypersurfaces as the null boundary of this local patch, which are themselves at a distance of order ΛL away from any point in it. The choice is also made in such a way that the boundaries are null geodesics.

Such a construction is shown in FIG. 1. It represents only the $t - n$ part and the other two space dimensions have been suppressed. $EFGH$ denotes the local patch around the point P . Let, $EF = GH = fL$ and $AC = BD = L$, where $f < 1$ but fL and L are both of the same order. This means that the local Minkowskian patch can be extended to cover the whole of \mathcal{D} . MPQ denotes the world line of an observer locally accelerating with acceleration a . One can draw null geodesics from any point in this patch, which intersects the causal diamond $ABCD$ surrounding it. PER is such a null geodesic. The affine distance along such null geodesics from any point in $EFGH$ to $ABCD$ is of the order of L . Measured in the scale of the local observer, this distance would be very large. In other words, for a local observer within $EFGH$, the boundary of the causal diamond \mathcal{D} is very far away. So \mathcal{D} can be taken as the null boundary of such an observer. Admittedly, such a choice of the null boundary is arbitrary to an extent. There is however an upper limit to the arbitrariness as the size of \mathcal{D} has to be much less than the scale fixed by the curvature of the spacetime. So two such choices of the null boundary cannot be removed from each other by a distance greater than the order L .

The velocity vector field of the accelerating observers, is denoted by t^μ , where $t^\mu t_\mu = -1$. Let, the unit vector along the direction of acceleration be denoted by n_μ . Such an observer is at rest in the accelerating frame \mathcal{A} . If we denote the coordinates in this frame by (x^0, x^1, x^2, x^3) , the energy flux 3-vector is T^{0i} , where $T_{\mu\nu}$ is the energy-momentum tensor. Now, in the frame \mathcal{A} , $t^a = 0$, where a denotes the spacelike indices.

Since one can neglect the curvature effects inside the causal diamond \mathcal{D} , a timelike Killing vector can be defined there. We further demand that the C.S. in A frame is such that there exists a timelike Killing vector, ξ^μ , parallel to t^μ . Also ξ^μ can be written as $(1, 0, 0, 0)$ and it satisfies the relation

$$\xi^\mu_{;\nu} n^\nu = a \xi^\mu. \quad (1)$$

A priori, it is not obvious whether such a condition could be made to satisfy around any spacetime point P . However, we are going to show later that this is indeed the case, i.e. such a construction is possible around any spacetime point P . It would also turn out that in this construction, the

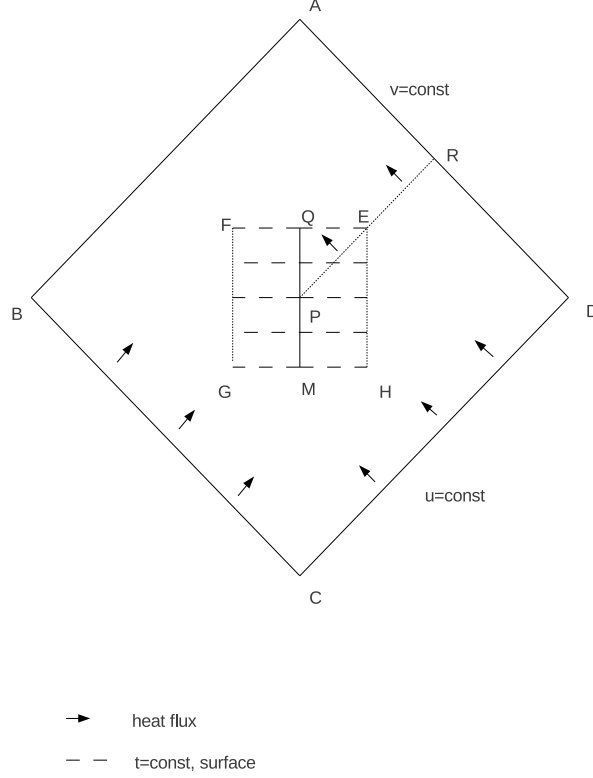


FIG. 1: Locally constructed Causal Diamond \mathcal{D} in the $t - n$ space

metric is of the same form as in the rest frame of any locally accelerating observer. To simplify the form of the metric, we are now going to make some further assumptions. First, we choose it to be of the form, $ds^2 = g_{00}(dx^0)^2 + dn^2 + \sigma_{AB}dx^A dx^B$, where A and B can run from 1 to 2. We also demand that $t^\alpha_{;\beta} e^\beta_A = 0$ where $A = 1, 2$. Two points should be noted here. Firstly, the direction corresponding to the vector $(0, dn, 0, 0)$ is being singled out. This is because the observer is accelerating along this direction. In other words, $d\mathbf{n}$ is chosen to be parallel to the acceleration vector \mathbf{a} . The second point is that we are also assuming that the velocity vector field does not change as we move along the other two spacelike directions within the local patch.

We shall now determine the form of the metric in the C.S. of A . This will also demonstrate that (1) can always be satisfied up to first order within the local patch that we have been considering. The acceleration vector \mathbf{a} is given by

$$t^\alpha_{;\beta} t^\beta = a^\alpha. \quad (2)$$

The norm of ξ is denoted as

$$-\xi^\mu \xi_\mu = \kappa_\xi^2 = \kappa. \quad (3)$$

Since t^α is parallel to the Killing vector ξ^α , we also have

$$t^\alpha = \frac{\xi^\alpha}{\kappa_\xi}. \quad (4)$$

Denoting the proper time along the trajectory of the accelerated observer by τ , we can write

$$\frac{d\xi_\mu}{d\tau} = \kappa_{\xi,\mu}, \quad (5)$$

where we have used the fact that t^μ is a unit vector and also the Killing equation for ξ . From the Killing equation, one also gets

$$\xi^\mu{}_{;\nu}\xi^\nu = \kappa_\xi\kappa_{\xi,\nu} \quad (6)$$

and using (6),

$$a = (\ln \kappa_\xi)_{,\alpha} n^\alpha = \frac{\partial}{\partial n}(\ln \kappa_\xi). \quad (7)$$

This fixes the g_{00} component of the metric. To fix the other components, we note that $\xi_{\alpha;\beta}n^\beta = a\xi_\alpha + \kappa_\xi t_{\alpha;\beta}n^\beta$. Then using (1), we get

$$t_{\alpha;\beta}n^\beta = 0. \quad (8)$$

This fixes the metric to be of the form,

$$ds^2 = -e^{2an} dt^2 + dn^2 + \sigma_{AB}(x^1, x^2) dx^A dx^B. \quad (9)$$

Since t^β and n^β are orthogonal to each other, using the Killing equation, it follows that

$$\xi_{\alpha;\beta}n^\beta = n_{\beta;\alpha}\xi^\beta. \quad (10)$$

From (10) and (1), one gets $a\xi_\alpha = n_{\beta;\alpha}\xi^\beta$. This equation implies

$$ag_{\alpha\beta} = n_{\beta;\alpha} + f(x^\mu)h_{\alpha\beta}, \quad (11)$$

where, $h_{\alpha\beta} = g_{\alpha\beta} + t_\alpha t_\beta$, is the projection operator on surfaces orthogonal to t_α and $f(x^\mu)$ is an arbitrary function of x^μ . To fix f , we take the trace on both the sides of (11) and obtain

$$4a = k + 3f, \quad (12)$$

where $k = n^\alpha{}_{;\alpha}$ is the extrinsic curvature scalar of the two dimensional hypersurface orthogonal to n_μ . To determine f , we contract (11) by n^μ . This gives $f = a$. Putting it back in (12), we have

$$k = a. \quad (13)$$

It is interesting to note that the acceleration and hence also the temperature seen by the accelerating observer comes out to be the extrinsic curvature scalar of the two dimensional spacelike hypersurface orthogonal to the direction of acceleration. Another point should also be mentioned here. Near the event horizon of a black hole and also near the Rindler horizon, the null vector normal to the horizon and the stationary Killing vector obeys a relation similar to the one between n_μ and the timelike Killing vector ξ^μ , as given by (1). However, at the horizon, the stationary Killing vector itself becomes a null vector normal to the horizon.

Now, we shall show that given a spacetime, locally it is always possible to construct such a C.S.. For this, let us first consider the null geodesics in the $t - n$ plane. They are given by

$$t = \mp \frac{1}{a}(e^{-an} - 1). \quad (14)$$

We define the null coordinates u and v by

$$u = -t + \frac{1}{a}(e^{-an} - 1); v = t + \frac{1}{a}(e^{-an} - 1). \quad (15)$$

Denoting the $t - n$ part of the metric by ds_{1+1}^2 , we have

$$ds_{1+1}^2 = e^{2an} du dv. \quad (16)$$

Now we are going to use the condition that the metric is being determined only in a local patch. Using the fact that $t \ll 1$ and $n \ll 1$ and ignoring the second order terms, one gets $u \approx -t - n$ and $v \approx t - n$. From the new expressions of u and v , it is possible to write $2n \approx -(u + v)$. We also note that within this scheme of approximation, (9) can be expressed as

$$ds^2 \approx -(1 + 2an)dt^2 + dn^2 + \sigma_{AB}(x^1, x^2)dx^A dx^B. \quad (17)$$

In what follows, we shall freely interchange the exponential functions of the variables u, v, t and n with their polynomial forms approximated up to the appropriate order in these variables. Such manipulations do not lead to any inconsistency, as long as we keep in mind that they represent the functions (like exponentials) only up to a given order in these variables.

One can then express (16) as $ds_{1+1}^2 \approx e^{-2a(u+v)} du dv$. This can be rewritten as

$$ds_{1+1}^2 \approx d\bar{u} d\bar{v}, \quad (18)$$

where

$$\bar{u} \approx \frac{e^{-au}}{a}; \bar{v} \approx \frac{e^{-av}}{a}. \quad (19)$$

|| The Killing vector field vanishes at the bifurcation surface for Rindler spacetime.

This brings the $t - n$ part of the metric to a Minkowskian form. As there is no constraint on the $\sigma_{AB}(x^1, x^2)$ part of the metric, it can always be brought to an Euclidean form. It should be noted here that in the above analysis, we have always made the approximations in such a way, so that the metric contains terms only up to first order. (19) gives the relation between the two sets of null coordinates locally, and they contain terms up to second order. (18) establishes two things. Firstly, we note that up to the first order, locally it is always possible to transform from the C.S. in the A frame defined by us to one in which the metric is Minkowskian and vice versa. As it is always possible to express the metric locally (i.e. up to the first order) in a Minkowskian form, this demonstrates that the assumptions made by us about the properties of the Killing vectors defined locally and so on are justified. The second point is that the relation between the null coordinates in the Minkowskian frame and the accelerating frame A can be expressed in a form similar to that between the Rindler C.S. and Minkowskian one up to second order in u and v .

B. Heat generation induced by Gravity and the derivation of Einstein equation

We are now in a position to proceed to the thermodynamic analysis. The basic idea is very simple. For a causal diamond constructed locally in the way described earlier, the increase in gravitational entropy is assumed to be proportional to the amount of increase in the area of the cross-section of the null geodesic congruences along the properly specified part of the null boundaries of the causal diamond. A certain fraction of the energy flux crossing this diamond is assumed to be converted into heat. The amount of heat generated and the increase of the entropy are then related via the relation, $dQ = Tds$. Plugging in the expressions for these quantities, the Einstein equation is expected to follow.

The most important point on which, the construction given here differs from that in Jacobson's paper [5], is in determining the amount of heat generated. In Jacobson's construction, the Rindler horizon constitutes a part of the boundary of the locally constructed causal patch. The energy flux passing through the horizon is then taken to be the amount of energy that gets transformed into heat. The justification for this choice comes from many facts. The fact that thermodynamic laws are satisfied by the Rindler-like horizon itself provides strong support for this [13]. Also, within the membrane paradigm, one finds further evidence in support of such a view. The horizon acts like an absolute screen for the accelerating observer, who cannot observe the degrees of freedom outside the horizon. So it makes sense to interpret the energy crossing the horizon as heat energy. But for the causal diamond that we are considering, the null boundaries do

not constitute an absolute screen in the sense a horizon does. It only demarcates an effective boundary of a local patch, where one can define a stationary Killing vector up to first order accuracy. So we need a new prescription or method to determine the increase in heat energy. Moreover, such a prescription should also be able to do two things. Firstly, it should be able to reproduce the Einstein equation. It should also be able to include the fact that the Rindler-like observer sees the energy passing through the horizon as heat. Such a prescription is proposed below.

In a locally accelerating frame, there exists a heat flux due to gravity which heats up the null surfaces within the local patch. Let, $\delta\xi$ be the change in the stationary Killing vector ξ , due to the non-inertial motion of the observer as one moves orthogonally across any null hypersurface within this patch. Then $\delta\xi$ gives rise to a heat flux given by $T_{\mu\nu}\delta\xi^\mu\mathbf{k}^\nu$. The total amount of heat generated in this way in a locally accelerating frame, can be computed using this prescription.

It is easily seen that this prescription gives the desired expression for the increase in heat energy in the case considered by Jacobson [5]. In that case, the amount of matter-energy crossing the horizon, that is converted into heat in the frame of the Rindler observer, can be determined by using it. The coordinate system of the accelerating observer, which is locally Rindler, breaks down at the causal boundary, i.e. the Rindler horizon. Hence, the change in the stationary Killing vector or the Boost Killing vector as one crosses the horizon is taken to be equal to ξ itself. This gives the standard expression for the increase in heat in this case.

With this, we are in a position to derive the Einstein equation. First, let us focus on the amount of energy that would be converted into heat in the locally accelerated frame according to the prescription outlined above. Let, $k^\mu = \frac{dx^\mu}{d\lambda}$ be the tangent vector to a null geodesic. Since both n^μ and n_μ are constant vector fields within \mathcal{D} , one can write

$$d\lambda = \frac{dn}{k^\mu n_\mu}, \quad (20)$$

where dn is the amount by which n changes along the null geodesic. This formula would be useful while performing integrations along null geodesics.

Now according to the above discussion, the heat flux is to be computed from the change in ξ as one moves across a null hypersurface orthogonally and also it is the null hypersurfaces that heat up in this process. So it would be convenient if we first decide on what are the null hypersurfaces that we have to take into account to determine the total heat increase as seen by the observer. From the form of the metric (9), it is clear that the Killing vector ξ does not change as we move within the (t, x^1, x^2) subspace. Hence we need to consider only the null hypersurfaces within the

(t, n) subspace. These are a series of $u = \text{const}$ and $v = \text{const}$ hypersurfaces, which crisscross each other within the causal diamond. Now, within \mathcal{D} , there is a change in ξ as we move across one such $u = \text{const}$ surface and also across one such $v = \text{const}$ surface. This means that we have to add the heat fluxes due to these two sets of null surfaces. Since each null hypersurface gets heated up by the resultant heat flux, we have to compute the total amount of increase in heat by multiplying this heat flux with the appropriate volume form for each set of null surfaces and then adding them up.

To compute the amount of heat increase, we can start from the null surface CD , which is a surface on which $u = \text{const}$. Now starting from this null surface, we want to move across other $u = \text{const}$ surfaces towards the center of \mathcal{D} , i.e. in the direction n decreases. For this purpose, we choose the null vector \mathbf{k}_u normal to $u = \text{const}$ hypersurfaces, where $k_u^\mu \approx -t^\mu - n^\mu$. The part of the heat flux generated due to the change of ξ across $u = \text{const}$ surfaces is given by $T_{\mu\nu}\delta_{\mathbf{k}_u}\xi^\mu$, where, $\delta_{\mathbf{k}_u}\xi^\mu = \xi^\mu{}_{;\nu}k_u^\nu\delta\lambda_u$. The total heat energy flux is given by, $h^\mu = T_{\mu\nu}\delta_{\mathbf{k}_u}\xi^\mu + T_{\mu\nu}\delta_{\mathbf{k}_v}\xi^\mu$, where, \mathbf{k}_v is the appropriate null vector normal to $v = \text{const}$ surfaces and $\delta_{\mathbf{k}_v}\xi^\mu = \xi^\mu{}_{;\nu}k_v^\nu\delta\lambda_v$. λ_u and λ_v are the affine parameters along the null geodesics, the tangent vectors to which are \mathbf{k}_u and \mathbf{k}_v respectively. To choose \mathbf{k}_v , we start from the null boundary DA and again move inwards from it. This fixes $k_v^\mu \approx t^\mu - n^\mu$. Any $u = \text{const}$ surface gets heated up due to the sum of the heat fluxes which are proportional to $\delta_{\mathbf{k}_u}\xi$ and $\delta_{\mathbf{k}_v}\xi$. The $v = \text{const}$ null surfaces heat up in the same way. One can find out the total amount of increase in heat energy by adding up the contributions of all these four parts.

They are given by

$$dQ_{uu} = \int T_{\mu\nu}(\delta_{\mathbf{k}_u}\xi^\mu)k_u^\nu d\lambda_u dA,$$

$$dQ_{vu} = \int T_{\mu\nu}(\delta_{\mathbf{k}_v}\xi^\mu)k_u^\nu d\lambda_u dA,$$

$$dQ_{uv} = \int T_{\mu\nu}(\delta_{\mathbf{k}_u}\xi^\mu)k_v^\nu d\lambda_v dA,$$

$$dQ_{vv} = \int T_{\mu\nu}(\delta_{\mathbf{k}_v}\xi^\mu)k_v^\nu d\lambda_v dA,$$

where, $\delta_{\mathbf{k}_u}\xi^\mu = \xi^\mu{}_{;\nu}k_u^\nu\delta\lambda_u$ and $\delta_{\mathbf{k}_v}\xi^\mu = \xi^\mu{}_{;\nu}k_v^\nu\delta\lambda_v$ and λ_u and λ_v are the affine parameters along the null geodesics, the tangent vectors to which are \mathbf{k}_u and \mathbf{k}_v respectively. Also dA is the area element of the cross section of the null hypersurfaces of the causal diamond. The volume element $\mathbf{k}d\lambda dA$ is being used as we want to determine the total heat energy transferred to the null surfaces orthogonal to \mathbf{k} within the causal diamond. From (20), we have $d\lambda_u = \frac{dn}{k_u^\mu n_\mu}$ and $d\lambda_v = \frac{dn}{k_v^\mu n_\mu}$ and also similar relations for $\delta\lambda_u$ and $\delta\lambda_v$. Using them, the total amount of increase in heat energy can

be expressed as

$$dQ \approx 4 \int T_{\mu\nu} (\delta_{\mathbf{n}} \xi^\mu) n^\nu dndA, \quad (21)$$

where the relations $k_u^\mu n_\mu \approx -1$ and $k_v^\mu n_\mu \approx -1$ have also been used. Using (1), and using n and δn interchangeably (since $n \ll 1$), one can express the amount of heat increase as

$$dQ \approx 4a \int_0^n T_{\mu\nu} \xi^\mu n^\nu ndndA. \quad (22)$$

One important point to note here is that the amount of the increase in heat energy depends upon the fact that it is the null surfaces that are being heated up. This is a characteristic property of the gravitational heating process that is being considered here. As seen here, it has physical consequences.

Now we consider the right hand side of the equation, $dQ = Tds$. The temperature T is taken to be proportional to the acceleration a . The justification for this is provided in the next section. Assuming the gravitational entropy to be holographic and taking a cue from black hole thermodynamics, the increase in entropy ds is taken to be proportional to the increase of the area of the cross section of the null boundary of the diamond. This null boundary can be thought of as being constituted of two wedges and we consider the area increase along one of the wedges. Our calculation of the increase of the area of the cross section closely follows the one performed by Jacobson [5]. However, there are some important points, on which our method differs from his. These would be pointed out as we go along.

The increase of the area of the cross-section of the null boundary of \mathcal{D} is calculated along the wedge CDA . We choose two null congruences along CD and DA . They are chosen in such a way that for both the congruences, θ and σ vanish at C . Now matter-energy passes through both the sides CD and DA of the null wedge. Hence the focusing or the defocussing effect of this matter on the null congruences would be such that along CD , the area increases and the expansion becomes zero at C . However, θ is negative along DA and hence the area of the congruence decreases along it. We assume that $ds \propto (\delta A_{CD} + \delta A_{DA})$, where δA_{CD} and δA_{DA} are the changes in the area of the null congruences along CD and DA respectively. Now one potential problem with this assumption is that ds might turn out to be negative. However, as we shall see, in this case, it turns out that although $\delta A_{CD} > 0$ and $\delta A_{DA} < 0$, $(\delta A_{CD} + \delta A_{DA}) > 0$. So at least this problem of decrease in entropy is avoided. The fact that the sign of the change in the area of the null congruences is opposite along the sides CD and DA implies that the change in this area along the boundary of the causal diamond does not increase monotonically with time in the locally accelerating frame.

Instead, it increases first till the time slices evolve up to the hypersurface BD and then start decreasing. This means that even though the entropy is proportional to the total change in the area of the null congruences along the null boundary, within the causal diamond itself, one cannot assign it to be proportional to the incremental change in area with time. In other words, using this prescription, one can only determine the total entropy increase in the causal diamond \mathcal{D} as a whole. This feature of the entropy as defined here contrasts strikingly with that of the horizon entropy, which increases monotonically, once a particular type of teleological boundary condition has been assumed. Using this prescription, one can only find out about the overall increase in entropy within the locally constructed causal diamond. We shall come back to this issue again in the final section of this paper.

The task of calculating the change in the area of the boundary, \mathcal{B} , in the way outlined above is straightforward. The change in the area is

$$\delta A = \int_{\mathcal{B}} \theta d\lambda dA, \quad (23)$$

where, λ is the affine parameter along the null geodesic congruence. Now the Raychaudhuri equation for the null congruence is given by,

$$\frac{d\theta}{d\lambda} = -\frac{1}{2}\theta^2 - \sigma^{\mu\nu}\sigma_{\mu\nu} - R_{\mu\nu}k^\mu k^\nu, \quad (24)$$

where, $\sigma_{\mu\nu}$ denotes the shear of the congruence and \mathbf{k} is a null vector tangent to the null geodesics. We choose the boundary in such a way that θ and σ vanishes at D . Neglecting the higher order terms, one gets from (24), $\theta = -\lambda R_{\mu\nu}k^\mu k^\nu$. Putting this back in (23), we have

$$\delta A = - \int_{\mathcal{B}} \lambda R_{\mu\nu}k^\mu k^\nu d\lambda dA. \quad (25)$$

The null vectors tangent to the sides CD and DA that we are going to use below are denoted by $\bar{\mathbf{k}}_{\mathbf{u}}$ and $\bar{\mathbf{k}}_{\mathbf{v}}$ respectively. They are given by the relations $\bar{k}_u^\mu \approx t^\mu + n^\mu$ and $\bar{k}_v^\mu \approx t^\mu - n^\mu$. The total area change along the boundary can be written as

$$\delta A = - \left(\int_{CD} \lambda_u R_{\mu\nu} \bar{k}_u^\mu \bar{k}_u^\nu d\lambda_u dA + \int_{DA} \lambda_v R_{\mu\nu} \bar{k}_v^\mu \bar{k}_v^\nu d\lambda_v dA \right). \quad (26)$$

It is important to recall here that λ_u increases monotonically along CD from a negative value at C to zero value at D , whereas λ_v increases monotonically along DA from zero value at D to a positive value at A . Let us assume that n increases along CD from zero value at C to n_{max} at D and then again decreases along DA , becoming zero at A . Then changing the variables from λ_u and λ_v to n , we get

$$\delta A = - \int_0^{n_{max}} (n - n_{max}) R_{\mu\nu} (\bar{k}_u^\mu \bar{k}_u^\nu - \bar{k}_v^\mu \bar{k}_v^\nu) dn dA. \quad (27)$$

Since $R_{\mu\nu}$ is a symmetric tensor, one may add a term $R_{\mu\nu}(k_u^\mu k_v^\nu - k_v^\mu k_u^\nu)$ in the integrand of the R.H.S. of (26), without changing anything. Then defining a variable, $x = n - n_{max}$, we get

$$\delta A = 4 \int_0^{-n_{max}} R_{\mu\nu} t^\mu n^\nu x dx dA. \quad (28)$$

Now ignoring second order terms in x in the integrand of (28),

$$\delta A = 4 \int_0^{n_{max}} R_{\mu\nu} t^\mu n^\nu n dn. \quad (29)$$

The upper limit of n in the n -integral of (22) is also n_{max} . Now assuming that $dQ = Tds$ holds, and using (22) and (29), one gets

$$T_{\mu\nu} \xi^\mu n^\nu = R_{\mu\nu} t^\mu n^\nu. \quad (30)$$

(30) can be written as $\kappa_\xi T_{\mu\nu} t^\mu n^\nu = R_{\mu\nu} t^\mu n^\nu$. Now neglecting the first order terms, one gets

$$T_{\mu\nu} t^\mu n^\nu = R_{\mu\nu} t^\mu n^\nu. \quad (31)$$

Since t^μ and n^μ are orthogonal and one can choose t^μ as one wishes, the following identity holds true,

$$T_{\mu\nu} + f(x^\alpha) g_{\mu\nu} = R_{\mu\nu} \quad (32)$$

It can be easily shown that if the energy-momentum tensor is assumed to be conserved, then it follows that, $f = -\frac{R}{2} + \Lambda$, where Λ is a constant and one ends up with the Einstein equation along with a cosmological constant term.

III. DETECTION OF THERMAL ENVIRONMENT BY A LOCALLY ACCELERATING OBSERVER

In this section, it is shown how, an observer at rest in a locally accelerating frame, would encounter a thermal environment. Before proceeding with the analysis, the notion of the locally accelerating observer has to be made more precise. Specifically, the key feature distinguishing any such observer is that the scales at which she could make a measurement is much smaller than the size of the local patch. It would also be assumed that the physics at this smaller scale would be local up to a very good approximation, i.e. it would not be influenced by physics at a scale, which is much larger than the size of the local patch. For the patch that we are considering, it is also possible to transform from the C.S. in which the accelerating observer is at rest to the C.S. in which the

inertial observer is at rest. Since local observers can observe physics at a much smaller scale than the size of this patch, it can be treated as the Minkowski spacetime as far as they are concerned. These considerations should hold in both these frames up to a very good approximation.

Now, it would be further assumed that within these approximations, the 'inertial' observer is going to see a state analogous to a vacuum state in the Minkowski spacetime in the inertial frame. So one has to construct such states. This can be done as follows. First, one focuses on a particular wave equation of a free field, like the wave equation for the massless Scalar field. Then one tries to build Fock states by taking into account only the short wavelength modes associated with the local patch or in other words, by excluding the wave modes which are of wavelength larger than the size of the local patch. In this approximate sense, one can talk about the different Fock states associated with this local patch. The assumption made here is that the inertial observer associated with this local patch would not detect any particles, i.e. the inertial observer finds the field to be in the vacuum state. In a similar way, it is also possible to construct the Fock states using the short wavelength modes associated with the local patch for the observers, who are accelerating. What will be shown here is that in this case, those accelerating observers find themselves in a thermal environment.

To deal with the wave modes, which are much smaller than the size of the local patch that we have been considering, let us define the quantities, $t_s = \Lambda t$, $n_s = \Lambda n$, $x_s^1 = \Lambda x^1$ and $x_s^2 = \Lambda x^2$. These are just the rescaled time and space coordinates. The rescaling factor is a constant Λ , chosen in such a way, that $\Lambda L \gg 1$, where L is the size of the patch. This can be thought of as introducing very small units of length, which the observer is using to measure length and time, i.e. the length resolution power of the local observers is much higher. The value of any quantity after such a rescaling of the coordinates is going to be denoted here by the suffix 's'. The value of the acceleration after such a rescaling becomes $a_s = \frac{a}{\Lambda}$. This means that in the rescaled units, $a_s \ll 1$. Apart from being multiplied by a constant factor, the metric retains the same form in the accelerated frame after the rescaling of the coordinates. We consider only the (t, n) part of the metric here. Apart from a constant factor, it can be written as

$$ds_{1+1}^2 = -e^{2a_s n_s} dt_s^2 + dn_s^2. \quad (33)$$

One can now define u_s and v_s , which are of exactly the same form as in (15) with only a_s , t_s and n_s replacing a , t and n respectively. Consequently, (33) can be written as $ds_{1+1}^2 = e^{2a_s n_s} du_s dv_s$ in the same way as (16) was deduced. It is important to note here that $a_s n_s = an$. Hence $a_s n_s \ll 1$.

In the same way as in (19), one can define \bar{u}_s and \bar{v}_s by the relations

$$\bar{u}_s \approx \frac{e^{-a_s u_s}}{a}; \bar{v}_s \approx \frac{e^{-a_s v_s}}{a}. \quad (34)$$

One can transform to the coordinates \bar{u}_s, \bar{v}_s and write the metric in a Minkowskian form,

$$ds_{1+1}^2 \approx d\bar{u}_s d\bar{v}_s. \quad (35)$$

With the coordinates rescaled, we now move on to the issue of constructing the Fock states for the local observers in the inertial and in the accelerating frames. For this purpose, we shall consider a free massless scalar field ϕ here. Within the scheme of approximation outlined above, one can expand $\phi(t_s, \vec{x}_s)$ in terms of the orthonormal modes as

$$\phi(t_s, \vec{x}_s) = \sum_{\mathbf{k}} [a_{\mathbf{k}} \phi_{\mathbf{k}}^{\bar{u}}(t_s, \vec{x}_s) + a_{\mathbf{k}}^\dagger \phi_{\mathbf{k}}^{\bar{u}}(t_s, \vec{x}_s)^*]. \quad (36)$$

In a similar way, in the accelerated frame also, $\phi(t_s, \vec{x}_s)$ can be expanded in terms of the orthonormal modes denoted by $\phi_{\mathbf{k}}^u(t_s, \vec{x}_s)$ as

$$\phi(t_s, \vec{x}_s) = \sum_{\mathbf{k}} [a_{\mathbf{k}} \phi_{\mathbf{k}}^u(t_s, \vec{x}_s) + a_{\mathbf{k}}^\dagger \phi_{\mathbf{k}}^u(t_s, \vec{x}_s)^*]. \quad (37)$$

The modes denoted by $\phi^{\bar{u}}$ are just the plane wave modes in the Minkowskian spacetime. However one needs to look at the modes denoted by ϕ^u in the accelerated frame more carefully. The wave equation for the massless scalar field ϕ can be written down in the accelerated frame as

$$-\partial_{t_s}^2 \phi - a_s \partial_{n_s} \phi \partial_{t_s} \phi + \partial_{n_s}^2 \phi + \sum_{i=1}^2 \partial_i^2 \phi = 0. \quad (38)$$

At this point, it is important to realise that the term proportional to a_s in the L.H.S. of (38) is going to be much smaller in magnitude than the other terms, since $a_s \ll 1$. Then one can treat this term as a perturbation to the free scalar field wave equation in Minkowski spacetime in the inertial frame. If the wave equation is then solved as a perturbation series order by order, then at the zeroeth order, (38) reduces to the standard wave equation in the inertial frame. The general solution to it can be written as a Fourier series of the plane wave modes. Even when one solves the equation in the first order, one can express the general solution as a Fourier series in terms of the same plane wave modes. Up to first order then, these plane wave modes constitute a complete orthonormal basis for (38). This fact is going to be used in the analysis that follows. That these plane wave modes from an orthonormal set is a well known fact and will be assumed as such.

Alternatively, this also follows from the fact that (38) can be expressed in the form

$$\frac{\partial^2 \phi}{\partial u_s \partial v_s} = 0, \quad (39)$$

in terms of the null coordinates u_s and v_s . An equation of the form (39) has plane wave modes as solutions [See [23]].

It should be emphasised here that it is only within a scheme of approximation, that these wave modes could be taken to form an eigenbasis in both the frames. The size of the patch being considered is of the order ΛL . So one can solve the massless Klein-Gordon equation with periodic boundary conditions in a box of size ΛL . Then the wavenumbers are discrete and given by $\frac{n\pi}{\Lambda L}$, where n belongs to the set of all positive integers. Now instead of taking the limit of ΛL going to infinity, we just consider the case when ΛL is very large. Then we can replace the discrete wavemodes by the continuous ones up to a very good approximation. It is only within this approximation that the operators a and a^\dagger are to be defined and the commutation relation between them would hold. In fact, in the commutation relation between a and a^\dagger , the right hand side can be thought of as a properly weighted sum of the discrete wave modes, which is a good approximation to a Dirac delta function, when ΛL is very large. However, one can also scale the x^μ s to get rid of the factor L and absorb it within a and a^\dagger as it is a constant factor. After that, only Λ , which is a very large number, remains in the denominator of the wave number. Then arguing in the same way as before, one can approximate this sum as a Dirac delta function.

Once the basis sets have been constructed in both the frames, we can ask the question, '*What state does the vacuum state in the inertial frame correspond to in the accelerated frame?*'. This can be answered by calculating the Bogulobov coefficients pertaining to this case. We shall now calculate these coefficients by closely following a similar calculation done by Visser *et al*[24]. A similar approach has also been followed in [25]. The Bogulobov coefficients are given by

$$\begin{aligned}\alpha(\omega, \bar{\omega}) &= (\phi^u(\omega; t, \vec{x}), \phi^{\bar{u}}(\bar{\omega}; t, \vec{x})) \\ &= -i \int d^3x \{ \phi^u(\omega; t, \vec{x}) \partial_t \phi^{\bar{u}}(\bar{\omega}; t, \vec{x})^* - \phi^{\bar{u}}(\bar{\omega}; t, \vec{x})^* \partial_t \phi^u(\omega; t, \vec{x}) \}\end{aligned}\quad (40)$$

and

$$\begin{aligned}\beta(\omega, \bar{\omega}) &= -(\phi^u(\omega; t, \vec{x}), \phi^{\bar{u}}(\bar{\omega}; t, \vec{x})^*) \\ &= -i \int d^3x \{ \phi^{\bar{u}}(\bar{\omega}; t, \vec{x}) \partial_t \phi^u(\omega; t, \vec{x}) - \phi^u(\omega; t, \vec{x}) \partial_t \phi^{\bar{u}}(\bar{\omega}; t, \vec{x}) \}.\end{aligned}\quad (41)$$

The integrals are over any arbitrary spacelike hypersurface within the local patch and extends at least till the boundary of the local patch. This can be thought of as the analogue of a spacelike hypersurface terminating at the spacelike infinity in a Minkowskian spacetime. In fact, as we have argued earlier, once we rescale the coordinates, the local patch can be treated as Minkowskian up

to a very good approximation. This implies that in our case also, the Bogulobov coefficients are Lorentz invariant and hence do not depend on the hypersurface on which they are evaluated. The frequencies ω and $\bar{\omega}$ correspond to the frequency of the wave modes in the accelerated frame and in the inertial frame respectively. In this case, it is convenient to evaluate them on a spacelike hypersurface arbitrarily close to the future null boundary of the local patch that we have been considering. This means that we have to integrate over both a $u = \text{constant}$ and a $v = \text{constant}$ surface. Since we have chosen the ϕ^u modes as the basis, the integrand vanishes on the $v = \text{constant}$ surface. Hence, for the coefficient β , we get

$$\beta(\omega, \bar{\omega}) \propto \frac{1}{\sqrt{\omega \bar{\omega}}} \int_{u_{s-}}^{u_{s+}} du_s \{ \exp[-i\omega u_s] \partial_{u_s} \exp[-i\bar{\omega} \bar{u}_s] - \exp[-i\bar{\omega} \bar{u}_s] \partial_{u_s} \exp[-i\omega u_s] \}, \quad (42)$$

where, $u_s \in \{u_{s-}, u_{s+}\}$ is the range of u_s . u_{s-} and u_{s+} are both of the order of ΛL .

The first term can be integrated by parts giving rise to a surface term proportional to $\exp[-i\omega u_s] \exp[-i\bar{\omega} \bar{u}_s]_{u_{s-}}^{u_{s+}}$, which would be discarded. Thus one obtains the relation

$$\beta(\omega, \bar{\omega}) \propto \sqrt{\frac{\omega}{\bar{\omega}}} \int_{u_{s-}}^{u_{s+}} du_s \{ \exp[-i\omega u_s] \exp[-i\bar{\omega} \bar{u}_s] \}. \quad (43)$$

Using the relation between \bar{u}_s and u_s , (43) can be written as

$$\beta(\omega, \bar{\omega}) \propto \sqrt{\frac{\omega}{\bar{\omega}}} \int_{u_{s-}}^{u_{s+}} du_s \exp \left[-i\omega u_s + i\bar{\omega} \frac{e^{-a_s u_s}}{a_s} \right]. \quad (44)$$

It is important to keep in mind that in (44), we have made use of the relation (34) between \bar{u}_s and u_s , where the exponential function is used only as an approximation. This can also be seen from the constraint, $a_s u_s \ll 1$, that has to be satisfied. This constraint implies that even though u_s becomes very large, $e^{-a_s u_s}$ would always have a value close to 1.

Now changing the variable from u_s to z , where

$$z = \exp(-a_s u_s), \quad (45)$$

(44) can be expressed as

$$\beta(u_s; \omega, \bar{\omega}) \propto \sqrt{\frac{\omega}{\bar{\omega}}} \frac{1}{a_s} \int_{z(u_{s-})}^{z(u_{s+})} dz z^{i\omega/a_s - 1} \exp(i\bar{\omega} \frac{z}{a_s}). \quad (46)$$

As has just been mentioned, the constraint forces both $z(u_{s-})$ and $z(u_{s+})$ to be close to 1. Let us define a variable, $z_1 = z - 1$. Then the exponent in the integrand can be expressed as $e^{i\bar{\omega}} \exp(i\bar{\omega} z_1)$. The first factor does not depend on z_1 and can be taken out of the integral. z_1 would range from 0 to $+\epsilon$ where $|\epsilon| \ll 1$. Now the factor $\frac{1}{a_s}$ in the exponent of the integrand is very large. This means that the frequency of the waveform in the integrand is very high. In other words, the waveform varies

very rapidly and so one can extend the range of integration from $\{0, +\epsilon\}$ to $\{-1, +\infty\}$. Extending the range of the integration contributes very little to the integral and can be neglected. One can go back to express the integral again in terms of z as the variable and the range of integration is now extended from 0 to $+\infty$. This is a crucial step in this derivation of the thermal spectra. As we shall see, the possibility of the extension of the integration range allows us to express the integral as a complete gamma function. This leads to a thermal spectrum. The extension of the range of the integration was possible because $a_s \ll 1$. Another point to remember is that to obtain the thermal spectra, we need not take the limit $\Lambda \rightarrow \infty$. The approximation to a thermal spectra is good enough for large values of Λ . This means that the rescaling of the coordinates in the local patch is directly responsible for the appearance of the familiar thermal weight factor. From a more physical perspective, the thermal nature is resulting from the fact that the observer sees only local physics, i.e. the wave modes that are being observed are localised within the patch we have been considering.

So then, (46) can be expressed as

$$\beta(u_s; \omega, \bar{\omega}) \propto \sqrt{\frac{\omega}{\bar{\omega}}} \frac{1}{a_s} \int_0^{+\infty} dz z^{i\omega/a_s - 1} \exp(i\bar{\omega} \frac{z}{a_s}). \quad (47)$$

The integral in (47) can be calculated as shown in [24]. After neglecting an irrelevant phase, one gets

$$\beta(u_s; \omega, \bar{\omega}) \propto \sqrt{\frac{\omega}{\bar{\omega}}} \frac{1}{a_s} \exp[-\pi\omega/(2a_s)] \Gamma(i\omega/a_s). \quad (48)$$

Using the formula for Gamma functions,

$$|\Gamma(ix)|^2 = \frac{\pi}{x \sinh(\pi x)}, \quad (49)$$

one gets

$$|\beta(u_s; \omega, \bar{\omega})|^2 \propto \frac{\omega}{\bar{\omega}} \frac{1}{a_s^2} \pi \frac{\exp(-\pi\omega/a_s)}{\omega/a_s \sinh(\pi\omega/a_s)}. \quad (50)$$

One can express this in the familiar form of Planck spectrum,

$$|\beta(u_s; \omega, \bar{\omega})|^2 \propto \frac{1}{a_s \bar{\omega}} \frac{1}{\exp(2\pi\omega/a_s) - 1}. \quad (51)$$

α can also be determined in a similar way.

(51) gives a temperature

$$k_B T = \frac{a_s}{2\pi}. \quad (52)$$

There is however one serious problem in our derivation of the Planck spectrum. As we see from (52), the temperature of the thermal radiation is proportional to a_s . Working in units such that $k_B = 1$, one gets the wavelength of the temperature of the order of $\frac{1}{a_s}$ or Λ . This is of the order of the size of the local patch, we have been considering. This means that the temperature of the radiation is not a good observable quantity that could be measured by the observer in the local patch. In fact, the local observer cannot detect whether the radiation is in thermal equilibrium or not. Hence it is not possible for the observer while remaining inside the local patch, to find out whether the state he is seeing is a thermal state or not. However, if the observer focuses on the localised modes or the higher energy modes, then he would find that these follow a Maxwell-Boltzmann spectrum, which is what the Planck spectrum tends to for the modes with energy much higher than the energy scale given by the temperature. So if *only* those localised wave modes are being measured by the observer, then he finds himself in a thermal environment. A possible way this could occur would be to use a detector at rest in the accelerating frame, coupled weakly to the massless scalar field. In this case, one finds the detector in equilibrium with a thermal environment, where the energy levels follow the Maxwell-Boltzmann distribution.

The above statement can be demonstrated by giving an example. One can consider a detector coupled to the massless scalar field. For the sake of concreteness, let us follow [23] and assume that the interaction Lagrangian is given by $cm(\tau_s)\phi[x_s(\tau_s)]$, where c is a small coupling constant and m is the detector's monopole moment operator. $x_s^\mu(\tau_s)$ gives the worldline, along which the detector moves and τ_s is the proper time along that worldline. We have already found out the state in which the field ϕ is in. Let us denote this state by $|T\rangle$. We assume further that initially the detector is in the ground state with energy E_0 . After interacting with the field ϕ , it would undergo a transition to an excited state with energy $E > E_0$, while the field would make a transition to an excited state $|\psi\rangle$. For small enough values of c , the amplitude of this transition, \mathcal{A} , is given by first order perturbation theory up to a good approximation.

$$\mathcal{A} = ic\langle E, \psi | \int_{-\infty}^{\infty} m(\tau_s)\phi[x_s(\tau_s)]d\tau_s | E_0, T \rangle \quad (53)$$

The equation for the time evolution of $m(\tau_s)$ is given by

$$m((\tau_s) = e^{iH_0\tau_s}m(0)e^{-iH_0\tau_s}, \quad (54)$$

where, $H_0|E\rangle = E|E\rangle$. Using this, (53) can be factorised and expressed as

$$\mathcal{A} = ic\langle E|m(0)|E_0\rangle \int_{-\infty}^{\infty} e^{i(E-E_0)\tau_s} \langle \psi | \phi(x_s) | T \rangle d\tau_s. \quad (55)$$

To find out whether the detector detects a thermal environment or not, one needs to calculate the transition probability, P of the state $|E_0, T\rangle$ to be excited to states with all possible values of energy E and ψ . This is given by

$$P = c^2 \sum_E |\langle E|m(0)|E_0\rangle|^2 \Xi(E - E_0), \quad (56)$$

where,

$$\Xi(E) = \int_{-\infty}^{\infty} d\tau_s \int_{-\infty}^{\infty} d\tau'_s e^{-iE(\tau_s - \tau'_s)} \langle T|\phi[x_s(\tau_s)]\phi[x_s(\tau'_s)]|T\rangle. \quad (57)$$

If now the detector is in equilibrium with the field ϕ and moves along some worldline $x_s(\tau_s)$, then for those worldlines,

$$\langle T|\phi[x_s(\tau_s)]\phi[x_s(\tau'_s)]|T\rangle = g(\Delta\tau_s), \quad (58)$$

where, $\Delta\tau_s = \tau_s - \tau'_s$. In this case, the number of quanta absorbed by the detector per unit proper time τ_s is constant. So the transition probability per unit proper time is given by,

$$\frac{dP}{d\tau_s} = c^2 \sum_E |\langle E|m(0)|E_0\rangle|^2 \int_{-\infty}^{\infty} d(\Delta\tau_s) e^{-i(E-E_0)\Delta\tau_s} \langle T|\phi[x_s(\tau_s)]\phi[x_s(\tau'_s)]|T\rangle. \quad (59)$$

Following [23], we can write this as

$$\langle T|\phi[x_s(\tau_s)]\phi[x_s(\tau'_s)]|T\rangle = \langle 0|\phi(x_s)\phi(x'_s)|0\rangle + \int d^3k \frac{n_{\mathbf{k}}}{|\mathbf{k}|} u_{\mathbf{k}}(x_s) u_{\mathbf{k}}^*(x'_s) + \int d^3k \frac{n_{\mathbf{k}}}{|\mathbf{k}|} u_{\mathbf{k}}^*(x_s) u_{\mathbf{k}}(x'_s). \quad (60)$$

The factor $\frac{n_{\mathbf{k}}}{|\mathbf{k}|}$ comes from evaluating the Bogulobov coefficients. From (60), the detector response function can be computed assuming that it is at rest in the accelerating frame and it turns out to be

$$\frac{\Xi(E)}{\mathcal{T}} = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} d(\Delta\tau_s) e^{-iE\Delta\tau_s} \int \frac{d^3k}{2\omega} \exp(i\omega\Delta\tau_s) \frac{n_{\mathbf{k}}}{|\mathbf{k}|}, \quad (61)$$

where, \mathcal{T} is the time period over which the detector is switched on. The R.H.S. of (61) can be computed and one gets

$$\frac{\Xi(E)}{\mathcal{T}} \propto n_E. \quad (62)$$

Here one uses the fact that for a massless scalar field, $\omega = k$. This in turn implies that

$$\frac{dP}{d\tau_s} \propto c^2 \sum_E |\langle E|m(0)|E_0\rangle|^2 e^{-\frac{2\pi(E-E_0)}{a_s}}. \quad (63)$$

From the Maxwell-Boltzmann exponential factor in the expression for the transition probability, it can be concluded that the detector is in a thermal equilibrium with the surroundings at a

temperature $\frac{a_s}{2\pi}$. Here we have taken $n_E \propto e^{-\frac{2\pi(E-E_0)}{a_s}}$ as according to the argument given earlier, the energy levels with lower values of energy would not be accessible to the detector. Hence the Planck factor goes over to the Maxwell-Boltzmann factor. It is also important to point out that in obtaining (59), the closure relation for a complete set of $|E\rangle$ s has been used. However, an assumption is made there that the range of E in that relation roughly satisfies a lower bound given by $\frac{1}{E} \sim \Lambda L$. This can be viewed as a manifestation of the fact that the detector is disentangled from outside the local patch. This requirement of course is in line with our demand that the local observer does not see the physics outside of this small patch.

The factor $\exp[-\beta_s E]$ is taken here as the signature of the thermal environment. However, T_s is very small. This makes β_s very large. So we have to check whether the weight factor $\exp[-\beta_s E]$ associated with an energy level E is going to be vanishingly small for all the excited states physically relevant for the locally accelerating observer. Physically, the more relevant quantity is the ratio of the weight factors for any two different energy states as seen by the local observer. Let us consider two energy states seen by the observer whose energies are removed from each other by an amount δE . Then the ratio or the relative weight of the higher energy state would be $\exp[-\beta_s \delta E]$. The smallest gap in energy that can be resolved by the observer would be of the order of $\frac{1}{\Lambda L}$. This gives the relative weight as $\exp[-\frac{\beta}{L}]$ or $\exp[-\frac{1}{aL}]$. While $aL \ll 1$, it is not vanishingly small and it is independent of the factor $\frac{1}{\Lambda}$.

IV. DISCUSSION

Jacobson's insight suggests that the Einstein equation could be a consequence of thermodynamics at a local level. This, however, works only in the local frame of those observers, who are accelerating at a rate much higher than the scale set by the curvature of the spacetime at that point. Here we explore the possibility of extending this viewpoint to include observers, who may accelerate locally at a rate much less than the scale set by the scale of the curvature of the spacetime at that point. This necessitates a key change in the way the thermodynamic formulation is set up. In the case Jacobson considers, the Rindler horizon constitutes a part of the boundary of the local patch containing the observer. The energy passing through the horizon is then inaccessible to the accelerating observer and can be interpreted as the amount of energy that gets converted into heat in the frame of that observer. However, if the acceleration of the observer is not large compared to the scale set by the curvature of the spacetime, then the Rindler horizon does not constitute any part of the boundary of the local patch containing the accelerating observer. In this case

then, it is not possible to interpret the energy passing through the horizon as the heat energy, as seen by the observer. For a thermodynamic derivation of the Einstein equation to be possible in such cases, one has to assume that energy is being converted into heat even though no horizon exists. This forces one to adopt a more general proposal for determining the amount of energy that should be converted to heat in the local frame of the accelerating observer. By now, it has been quite convincingly demonstrated[6] [17], [20] that the null surfaces have some properties that are closely related to the thermodynamics of a system. This suggests the possibility, that for the locally accelerating observer, some fraction of the matter-energy passing through the null surfaces is converted into heat. Apart from this, it is also assumed that the amount of heat generated in this way, depends on the non-inertial motion of the observer. This links the change in the timelike Killing vector across the null boundaries of the local patch to the amount of energy converted into heat. It allows one to view heat conversion in the case of matter falling through the horizon as a special case of a more general process of gravitational conversion of matter-energy into heat energy. This is the main new result of this paper.

It is interesting to make an estimate of the fraction of matter-energy that gets converted into heat according to this proposal. For observers locally accelerating at a rate much smaller than the scale set by the curvature of the spacetime, this factor is in fact quite small. A rough estimate of this conversion factor can be made by comparing the heat flux through the locally constructed two dimensional surface perpendicular to n_μ with the total energy flux through that same surface. The ratio of these two fluxes turns out to be of the order of $\sim an$. As has been argued earlier in this paper, $n \ll \frac{1}{\sqrt{Riemann}}$ and $an \ll 1$, for not so large values of the acceleration. For the local observer, the more relevant quantity is the fraction of the matter-energy that gets converted into heat per unit length of the local patch. This is proportional to the acceleration of that observer. In the rescaled coordinates of the localised observer, this acceleration is given by a_s . This is going to be very small due to the presence of the rescaling factor $\frac{1}{\Lambda}$. So the amount of heat generated would be rather tiny for the locally accelerating observer to measure. In the limit of Λ going to infinity, the conversion factor goes to zero.

Another important feature of the heat generated in this way is that the amount of increase in heat as perceived by the local observer would depend on the curvature of the spacetime. This becomes clear if we look at the factor of conversion a_s more closely. The factor that matters most here is $\frac{1}{\Lambda}$. Now, Λ is of the order of the ratio of the size of the local patch to the smallest possible lengthscale of the localised wave modes that the locally accelerating observer could measure. If we denote the length of the size of the local patch by L , then $\Lambda \sim \frac{L}{u_{n_s}}$, where u_{n_s} is the unit of length

used by the local observer and it is of the same order as that of the lengthscale of the localised wave modes. Now if we assume that the magnitude of u_{n_s} remains roughly of the same order, then the dependence of Λ on the curvature can be expressed as $\Lambda \ll \frac{1}{(\sqrt{Riemann})u_{n_s}}$. This means that the conversion factor is of the order of $\frac{1}{\Lambda} \gg (\sqrt{Riemann})u_{n_s}$. So, the observer accelerating locally would find the amount of the energy that is being converted into heat to be greater, if the components of the Riemann curvature tensor at that point in spacetime is greater in magnitude. This suggests that the local observer perceives the amount of heat generated by gravity to be greater, when gravity is stronger.

It is also important to note that the way ds was calculated for the causal diamond \mathcal{D} , tells us only about the total entropy increase. But it does not tell us how ds is changing with time within the local patch. This is not surprising since we solved the Raychaudhuri equation with a different boundary condition instead of the standard teleological condition used to prove the second law for the horizons. It is not clear at this moment, how one can prove some version of the second law of thermodynamics in this case.

Finally, we also demonstrate here that a locally accelerating observer encounters a thermal environment with a temperature proportional to the value of her acceleration. This holds provided one is observing local physics only. This is reminiscent of the Unruh effect and also technically the calculation is very similar. However, there is a very important difference. In the case considered here, the locally accelerating observer does not observe thermal radiation at the temperature a_s for the obvious reason that the lengthscale corresponding to this temperature is much greater than the size of the local patch containing her. But any detector in her rest frame, interacting with much more localised modes of the radiation, would detect a thermal environment, The energy distribution in this case is found to be given by the Maxwell-Boltzmann distribution. This also means that the state seen by the observer need not be a thermal one. This is different from the Unruh effect, in which case the state seen by the Unruh observer is thermal. But in the case dealt with here, it is not possible for local observers to stay within the local patch and also determine whether the state seen by them is thermal or not. All they can do is determine the weights of the different energy-states (these are different from the quantum state of the field) and that comes out to be $\exp[-\beta_s E]$. This justifies the assignment of a temperature proportional to a in the locally accelerating frame. A point can be made that how much significance should one attach to this result given that the state seen by the local observer may not be thermal. The point of view taken here is that the environment is indeed thermal as far as the local observers are concerned. Also the fact that it is possible to derive the Einstein equation from thermodynamic relations in this case lends some support to it. Taken

together, they give credence to the view that for any locally accelerating observer, the dynamics of gravity can be viewed as a thermodynamic process.

The temperature measured by the observer goes to zero as the acceleration a vanishes. The observer for whom $a = 0$, is falling freely in that local patch. So, it is possible for any observer who is not falling freely to view the Einstein equation locally as a manifestation of thermodynamic changes in a system. In this sense, the result presented here is a generalisation of Jacobson's derivation of Einstein equation from thermodynamics to the case of any non-freely falling observer. A freely falling observer in that same local patch does not detect any temperature or the heating up of null surfaces by gravity. This is similar to the difference that exists between a Rindler-like observer and an inertial observer or between an observer freely falling through the black hole horizon vis-a-vis one who remains outside. A final remark is in order here. Due to the equivalence principle, the above considerations would apply to any observer in a locally constant gravitational field as well.

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